例題 48

(2) 別解

$b_n=\frac{1}{3}a_{n-1}+\frac{1}{3}c_{n-1}$，$d_n=\frac{1}{3}a_{n-1}+\frac{1}{3}c_{n-1}$ より，$b_n+d_n=\frac{2}{3}(a_{n-1}+c_{n-1})$

よって，$a_{n+1}+c_{n+1}=\frac{1}{3}(a_{n+1}+c_{n+1})+\frac{2}{3}(a_{n-1}+c_{n-1})$

(3)

$a_{n+1}+c_{n+1}=\frac{1}{3}(a_{n+1}+c_{n+1})+\frac{2}{3}(a_{n-1}+c_{n-1})$ について，$A_n=a_n+c_n$ とおくと，

$A_{n+1}=\frac{1}{3}A_n+\frac{2}{3}A_{n-1}$ より，$A_{n+1}-A_n=-\frac{2}{3}(A_n-A_{n-1})$，$A_{n+1}+\frac{2}{3}A_n=A_n+\frac{2}{3}A_{n-1}$

よって，

$$\begin{aligned}A_{n+1}-A_n&=-\frac{2}{3}(A_n-A_{n-1})\\&=\left(-\frac{2}{3}\right)^n(A_1-A_0)\\&=\left(-\frac{2}{3}\right)^n\{(a_1+c_1)-(a_0+c_0)\}\\&=\left(-\frac{2}{3}\right)^n\left\{\left(0+\frac{1}{3}\right)-(1+0)\right\}\\&=\left(-\frac{2}{3}\right)^{n+1}\end{aligned}$$

$$\begin{aligned}A_{n+1}+\frac{2}{3}A_n=A_n+\frac{2}{3}A_{n-1}&\\&=A_1+\frac{2}{3}A_0\\&=(a_1+c_1)+\frac{2}{3}(a_0+c_0)\\&=\left(0+\frac{1}{3}\right)+\frac{2}{3}(1+0)\\&=1\end{aligned}$$

これより，$\frac{5}{3}A_n=1-\left(-\frac{2}{3}\right)^{n+1}$

よって，$A_n=\frac{3}{5}\left\{1-\left(-\frac{2}{3}\right)^{n+1}\right\}$ すなわち $a_n+c_n=\frac{3}{5}\left\{1-\left(-\frac{2}{3}\right)^{n+1}\right\}=\frac{2}{5}\left(-\frac{2}{3}\right)^n+\frac{3}{5}$